Resistivity Image Reconstruction using *J*-Substitution Algorithm for MREIT

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Abstract-**We describe a new resistivity image reconstruction algorithm called *J*-substitution algorithm. It utilizes internal current density data measured by MRI technique with current injection. Computer simulations show that Magnetic Resonance Electrical Impedance Tomography (MREIT) system using *J*-substitution algorithm can produce high-resolution static resistivity images of a subject.**

Keywords: **EIT, MREIT, *J*-substitution algorithm, resistivity image**

I. Introduction

In Electrical Impedance Tomography (EIT), we try to reconstruct a cross-sectional resistivity (or conductivity) image of a subject using boundary voltage and current measurements [1], [2]. The static image reconstruction becomes a severely ill-posed inverse problem due to the low sensitivity of boundary measurements to any changes of internal tissue resistivity values. It also suffers from modeling errors in capturing the boundary shape of the subject for the construction of a computer model.

Lately, a new MRI technique has been developed for the measurements of the internal current density distribution [3]-[6]. While we inject current through electrodes attached on the boundary of a subject, MR images are obtained using a spin-echo sequence synchronized with current pulses. Since the injected current generates small magnetic field, these MR images include the effects of injection current as perturbation in the phase of images. Given MR images with phase perturbation, we can compute the magnitude of current density at each pixel of the MR image.

We propose utilizing internal current density data to reconstruct static cross-sectional resistivity images. We call this new imaging method MREIT (Magnetic Resonance Electrical Impedance Tomography). Since MR images of a subject are available, we can minimize the modeling error in boundary shape of the subject. Furthermore, the internal current density data eliminate the severe sensitivity problem in conventional EIT image reconstruction. The major disadvantage of MREIT is the requirement of the expensive MRI system. However, since MRI is becoming more common in modern clinical situation, the advantage of getting accurate high-resolution resistivity images will overcome the cost problem.

In 1994, a new EIT image reconstruction algorithm was proposed using internal current density distribution measured by MRI technique [7]. By adjusting resistivity distribution of a computer model of a subject, they tried to minimize the error between the measured current density from the subject and the computed current density of the model using the finite element method. However, the algorithm did not utilize the internal current density data effectively resulting in a poor spatial resolution and convergence characteristics. In this paper, we assume that internal current density distribution is available from an MRI system including added current injection capability. We propose a new static resistivity image reconstruction algorithm called *J*-substitution algorithm. Computer simulations will show that accurate high-resolution static resistivity imaging with improved convergence characteristics is possible.

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II. Methodology

A. Problem Definition

Let Ω denote two-dimensional cross-section of an electrically conducting body with positive resistivity distribution denoted by r^*. Assume we measure the current density $\boldsymbol{J}$ in Ω due to an injection current I through electrodes attached on the boundary $\partial\Omega$. The inverse problem is to reconstruct r^* from the known data I, $J = |\boldsymbol{J}|$, and physical laws of electromagnetics. In this paper, we use only the magnitude of current density.

B. J-Substitution Algorithm

We construct a mathematical model of the body Ω with the same geometrical shape. For any given resistivity r of the model, the corresponding voltage V_r satisfies the boundary value problem

$$\begin{aligned} &\nabla\cdot\left(\frac{1}{r}\nabla V_r\right)=0 \quad \text{in } \Omega \\ &\frac{1}{r}\frac{\partial V_r}{\partial n}=j_I \quad \text{on } \partial\Omega \end{aligned} \tag{1}$$

where j_I is the current density at the boundary $\partial\Omega$ and n denotes the unit outward normal vector at the boundary $\partial\Omega$.

Related to the boundary value problem in Eq. (1), we introduce the following cost functional $\Psi(r)$:

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$$\Psi(r) := \int_\Omega \left| J^*(r) - \frac{1}{r(r)} E_r(r) \right|^2 dr \qquad (2)$$

where J^* is the magnitude of the observed interior current density and $E_r(r) = \left|\nabla V_r(r)\right|$ is the magnitude of the calculated electric field intensity obtained by solving Eq. (1) for a given r. After discretization of the model into N cells, we obtain the following resistivity update strategy from the minimization of the functional in Eq. (2).

$$\frac{1}{r_k} = s_k \Leftarrow \frac{J^*(r_k)}{E_r(r_k)} \quad \text{for } k = 0, \cdots, N-1 \qquad (3)$$

where r_k, s_k, and r_k are resistivity, conductivity, and center of the k-th cell, respectively. Here, we assume that resistivity is uniform within each cell. Kwon *et al.* describe the derivation of Eq. (3) in detail [8].

From the consideration on the uniqueness of the solution, Kwon *et al.* showed that we must use at least two different injection currents [8]. In this paper, we assume that 4 electrodes are attached on the top, bottom, left, and right side of the subject. Therefore, the current density J^1 is obtained due to the injection current between two electrodes at the top and bottom. J^2 is similarly obtained between the other pair of electrodes,

In *J*-substitution algorithm, two current density measurements, J^1 and J^2 are used as J^* in Eq. (3). We assume an initial guess on r^* as r^0 and iteratively solve Eq. (1) for r^p where p is the iteration number. In each iteration, the resistivity distribution r is updated using Eq. (3) for each cell or pixel.

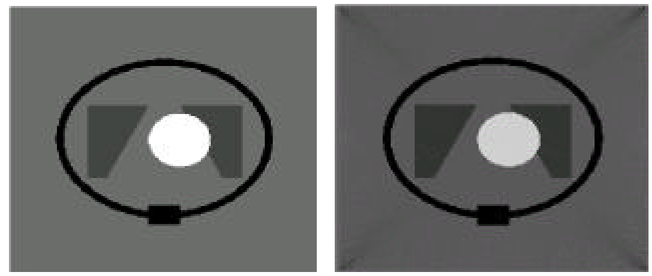
III. Results

A. Toy Model

Fig. 1(a) shows a target model called Toy model with 256×256 pixels. It consists of four anomalies with different resistivity values. The relative resistivity values with respect to the background are 0.25 in the white disk, 2 in two gray trapezoids, and 5 in the dark ring. J^1 and J^2 in Fig. 2(a) and (b) are simulated current density measurements with 5% random noise. After 20 iterations with the homogeneous initial guess, we obtain the reconstructed resistivity image shown in Fig. 1(b).

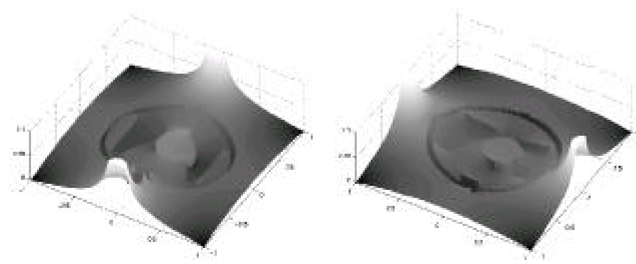
B. Realistic Model

The second model is introduced in order to show the feasibility of *J*-substitution algorithm in a very complicated realistic case. Because a real resistivity distribution of the human body is not available, we bravely imagine that the resistivity of each pixel is proportional to the pixel value of a CT image. Fig. 3(a) shows a realistic target model with 256×256 pixels. Fig. 5(a) and (b) show J^1 and J^2 which are simulated current density measurements with 3% random noise. The reconstructed resistivity image using the homogeneous initial guess is shown in Fig. 3(b) after 50 iterations.



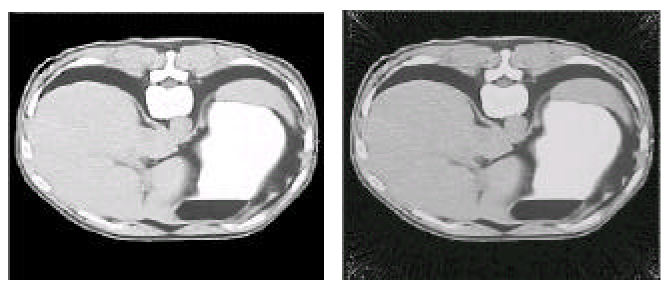
(a) (b)

Fig. 1. (a) Toy model and (b) reconstructed resistivity image.



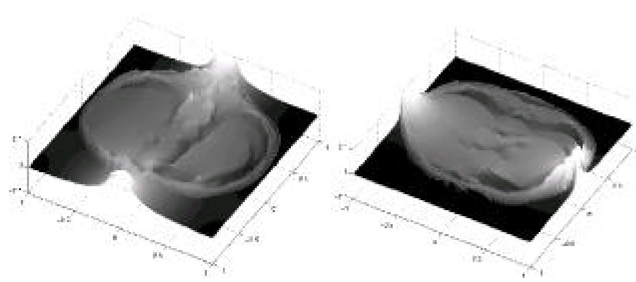
(a) (b)

Fig. 2. Simulated current density measurements of toy model with 5% noise. (a) J^1 and (b) J^2.



(a) (b)

Fig. 3. (a) Realistic model and (b) reconstructed resistivity image.



(a) (b)

Fig. 4. Simulated current density measurements of realistic model with 3% noise. (a) J^1 and (b) J^2.

IV. DISCUSSION

MREIT can solve many technical problems in conventional EIT. In static EIT imaging, we usually use 32 or more electrodes to achieve 5% spatial resolution at most. MREIT requires at least 4 electrodes, which is fewer than needed in EIT. This avoids the cumbersome electrode attachment procedure in EIT. In MREIT, we can easily obtain the boundary shape of the subject since MR images are available. This eliminates the problem related with modeling error. The severe sensitivity problem inherent in EIT is removed by utilizing internal current density data. MREIT takes the advantage of MRI as structural imaging modality and provides accurate tissue resistivity images that no other medical imaging modality can do.

When two pairs of electrodes are used as discussed in our paper, the magnitude of the current density is sufficient to guarantee the uniqueness of the reconstructed resistivity image [9]. Therefore, in this paper, we used only the magnitude of the current density vector. We will study the noise characteristics of the current density measurements and expand the algorithm to include the direction as well if it is helpful. However, we speculate that using only the magnitude information would be enough against noisy current density measurements.

Based on the simulation studies described in this paper, our algorithm shows convergence characteristics good enough for most highly complicated resistivity distributions that we can face in medical applications [8]. We also observed that any reasonable initial guesses make the algorithm convergent to the degree needed for successful image reconstructions [8]. As a part of our future works, we will rigorously study the mathematical analysis of the convergence.

The *J*-substitution algorithm described in this paper does not require nonlinear least squares optimization such as Newton-type algorithm that is computationally expensive. Since our algorithm requires simple substitutions, it only needs a fast forward solver for the solution of a linear system of equation. We may use the multi-grid method for cell-centered finite difference scheme or finite element method with sparse matrix techniques to accommodate any irregular boundary. In our future works, we will investigate the possibilities of improvement to seek for the ultimate performance of *J*-substitution algorithm.

We can construct an MREIT system by adding a constant current source, switching circuit, and at least 4 electrodes to any conventional MRI system. We are constructing a prototype of MREIT system based on 0.3 T laboratory MRI system with bore diameter of 25cm. We construct a phantom with different resistivity values that we can rotate in the magnet for measuring internal current density data. We expect the major problem in MREIT is the accuracy in the measured current density. Future studies will include the optimal MR imaging technique for accurate measurements of internal current density. We also need to study three-dimensional MREIT image reconstruction.

V. CONCLUSION

We proposed a new imaging technique, MREIT using *J*-substitution algorithm. Given reliable internal current density measurement technique, MREIT can provide clinically useful cross-sectional resistivity images of a subject. Even though it requires an expensive MRI system, it solves many technical problems in conventional static EIT imaging. Computer simulations show that MREIT produces high-resolution resistivity images. With future improvements in the construction of MREIT system including hardware and software, it will visualize totally new physiological information of resistivity distribution that no other imaging modality can provide. Accurate tissue resistivity values will be valuable in many biomedical application areas, especially where electric energy is utilized for diagnosis and treatment.

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